Algoritmos de Ordenação

Profa. Sheila Morais de Almeida

DAINF-UTFPR-PG

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Este material é preparado usando como referências os textos dos seguintes livros.

Análise de Algoritmos

Vamos analisar o Algoritmo Ordenação por Inserção.

Quantas operações de comparação são executadas?

Algoritmo Ordenação por Inserção
Entrada: vetor $v[1..n]$ de inteiros; n número de elementos no vetor

```
Para i de 2 a n faça:
    j ← i − 1
    Enquanto j > 0 && v[i] < v[j] faça:
        j ← j − 1
    t ← v[i]
    Para k de i − 1 a j + 1 faça:
        v[k + 1] ← v[k]
    v[j + 1] ← t
```

Análise de Algoritmos

Para o Algoritmo de Ordenação por Inserção, apresente uma instância

- de melhor caso;
- de pior caso;
- e de caso médio.

Algoritmo Ordenação por Inserção
Entrada: vetor $v[1..n]$ de inteiros; n número de elementos no vetor

Para i de 2 a n **faça:**
$j \leftarrow i - 1$
Enquanto $j > 0$ && $v[i] < v[j]$ **faça:**
$j \leftarrow j - 1$
$t \leftarrow v[i]$
Para k de $i - 1$ a $j + 1$ **faça:**
$v[k + 1] \leftarrow v[k]$
$v[j + 1] \leftarrow t$

Para o Algoritmo de Ordenação por Inserção, apresente uma instância

- de melhor caso: vetor em ordem crescente.
- de pior caso: vetor em ordem decrescente.
- e de caso médio: quando $v[i]$ é considerado, cada número de $v[0]$ a $v[i-1]$ tem 50% de chance de ser maior que $v[i]$ e 50% de chance de ser menor que $v[i]$. Para um exemplo, podemos supor:
  - os números em posições de 0 a $\lfloor \frac{i}{2} \rfloor$ são menores que $v[i]$ e
  - os números em posições de $\lfloor \frac{i}{2} \rfloor + 1$ a $i-1$ são maiores que $v[i]$.

Para o Algoritmo de Ordenação por Inserção, apresente uma instância

- de melhor caso: vetor em ordem crescente.
- de pior caso: vetor em ordem decrescente.
- e de caso médio: quando $v[i]$ é considerado, cada número de $v[0]$ a $v[i-1]$ tem 50% de chance de ser maior que $v[i]$ e 50% de chance de ser menor que $v[i]$. Para um exemplo, podemos supor:
  - os números em posições de 0 a $\lfloor \frac{i}{2} \rfloor$ são menores que $v[i]$ e
  - os números em posições de $\lfloor \frac{i}{2} \rfloor + 1$ a $i-1$ são maiores que $v[i]$.

  Ex.: 90 12 18 83 24 71 67 35 58 40.

Análise de complexidade de tempo no pior caso

Considere o Algoritmo de Ordenação por Inserção:

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    Para k de i − 1 a j + 1 faça:
        v[k + 1] ← v[k]
    v[j + 1] ← t
```

Análise de complexidade de tempo no pior caso

Pergunta: Quantas instruções básicas do modelo computacional RAM (operações aritméticas básicas, atribuições e comparações) são executadas pelo Algoritmo de Ordenação por Inserção, considerando uma entrada de tamanho n de pior caso?

Análise de complexidade de tempo no pior caso

Considere o Algoritmo de Ordenação por Inserção:

Algoritmo Ordenação por Inserção
Entrada: vetor $v[1..n]$ de inteiros; n número de elementos no vetor

Para i de 2 a n **faça:** $2 + 3(n-1) = 3n - 1$
  $j \leftarrow i - 1$ $2(n-1) = 2n - 2$
  Enquanto $j > 0$ && $v[i] < v[j]$ **faça:** $2\sum_{c=1}^{n-1} c + (n-1)^*$
    $j \leftarrow j - 1$ $2\sum_{c=1}^{n-1} c = n^2 - n$
  $t \leftarrow v[i]$ $n - 1$
  Para k de $i - 1$ a $j + 1$ **faça:** $4\frac{n(n-1)}{2} + 4(n-1)^\dagger$
    $v[k+1] \leftarrow v[k]$ $n(n-1) = n^2 - n$
  $v[j+1] \leftarrow t$ $2(n-1) = 2n - 2$

$^*2\sum_{c=1}^{n-1} c + (n-1) = n(n-1) + (n-1) = (n+1)(n-1) = n^2 - 1$

$^\dagger 4\frac{n(n-1)}{2} + 4(n-1) = 2n^2 + 2n - 4$

Análise de complexidade de tempo no pior caso

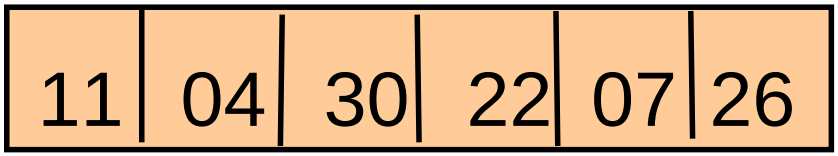
Considere o Algoritmo de Ordenação por Inserção:

Algoritmo Ordenação por Inserção
Entrada: vetor $v[1..n]$ de inteiros; n número de elementos no vetor

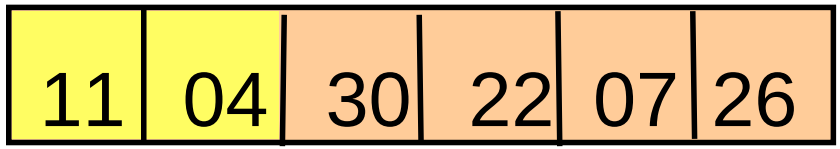
Para i de 2 a n **faça:** $2 + 3(n-1) = 3n - 1$
  $j \leftarrow i - 1$ $2(n-1) = 2n - 2$
  Enquanto $j > 0$ && $v[i] < v[j]$ **faça:** $n^2 - 1$
    $j \leftarrow j - 1$ $2\sum_{c=1}^{n-1} c = n^2 - n$
  $t \leftarrow v[i]$ $n - 1$
  Para k de $i - 1$ a $j + 1$ **faça:** $2n^2 + 2n - 4$
    $v[k+1] \leftarrow v[k]$ $n(n-1) = n^2 - n$
  $v[j+1] \leftarrow t$ $2(n-1) = 2n - 2$

Total: $5n^2 + 8n - 11$

Bubble Sort



Bubble Sort



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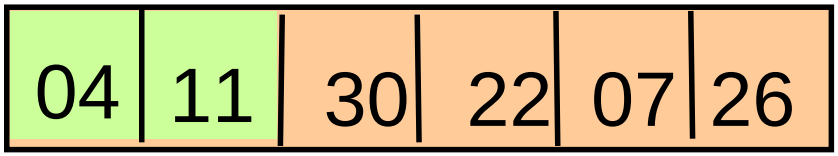

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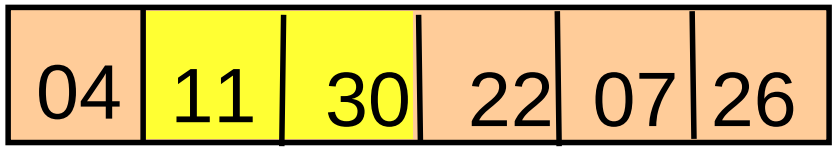

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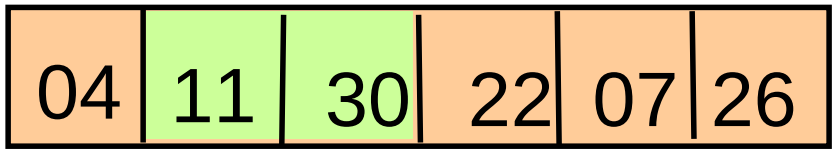

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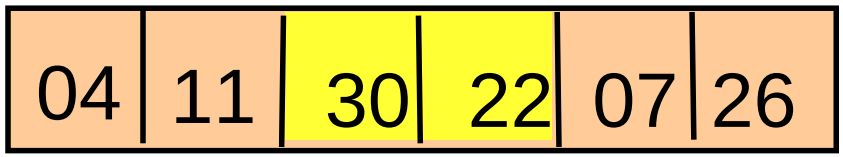

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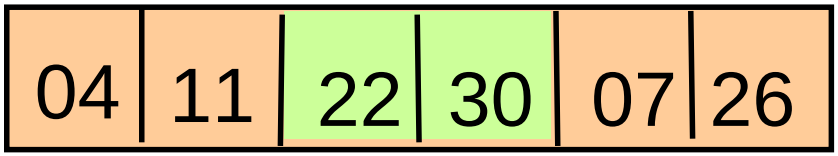

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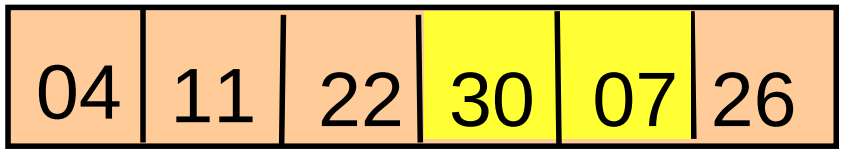

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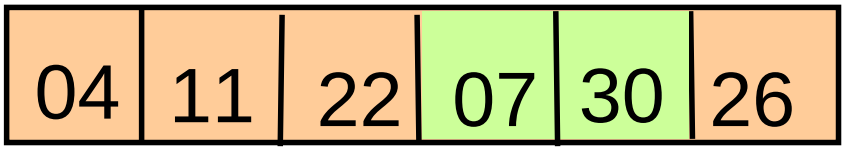

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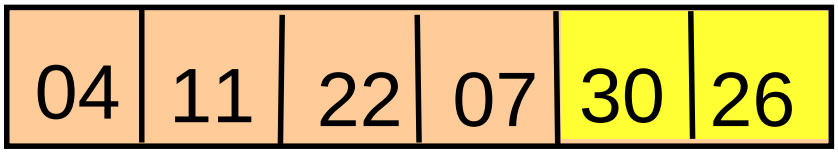

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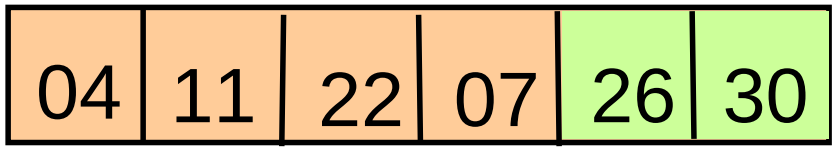

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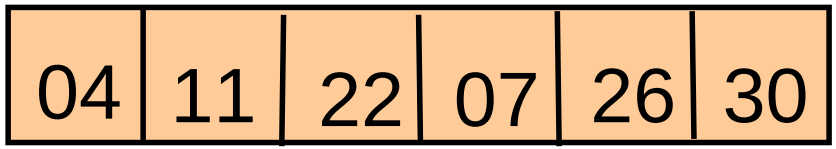

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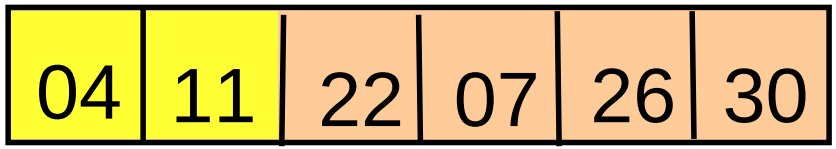

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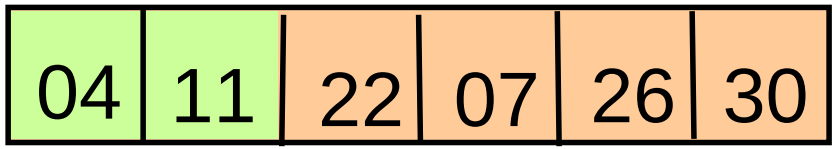

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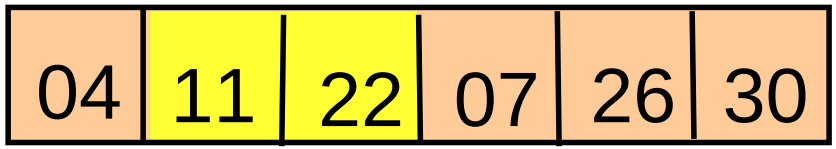

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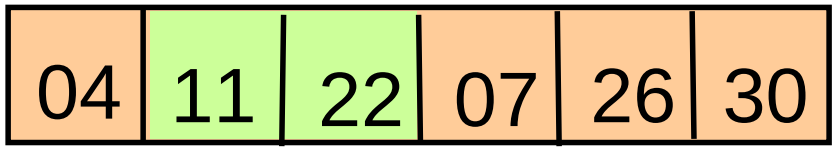

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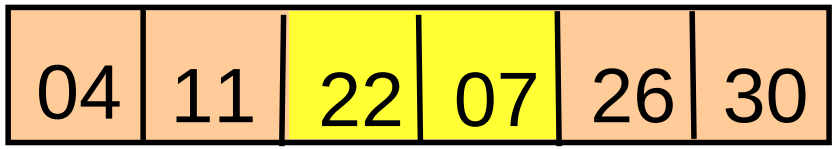

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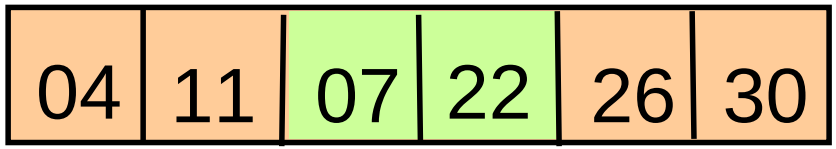

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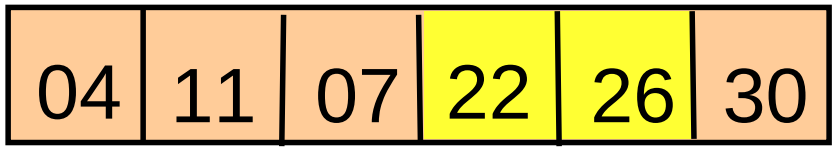

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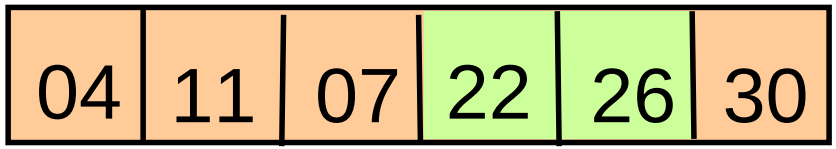

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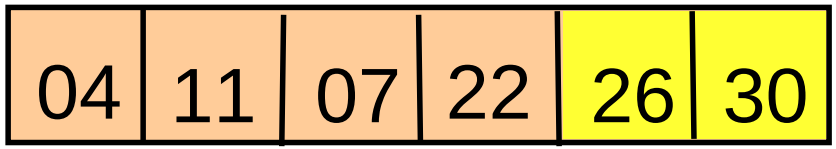

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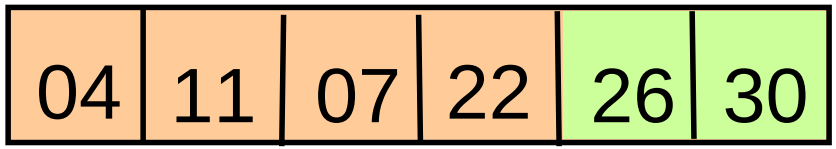

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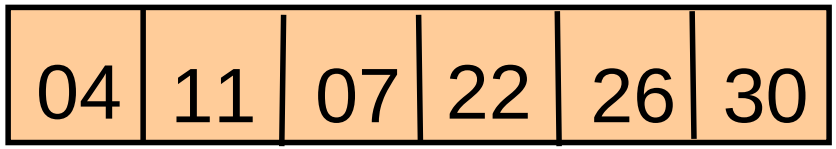

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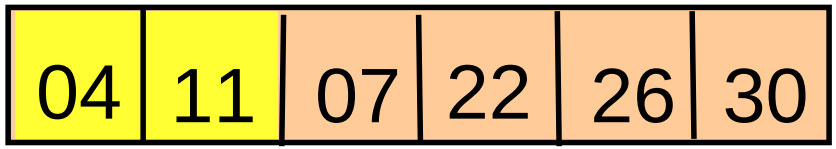

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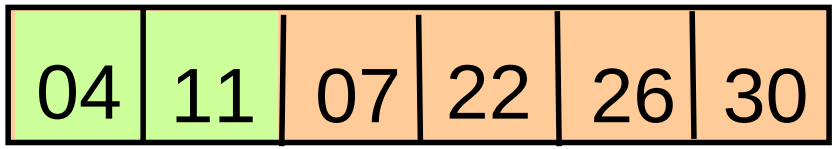

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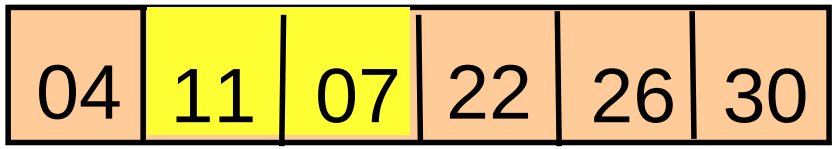

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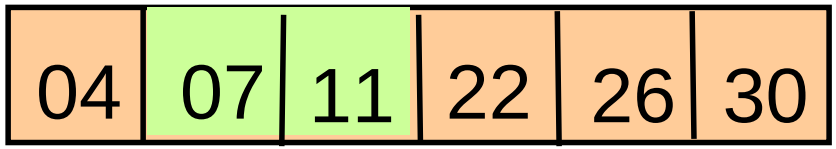

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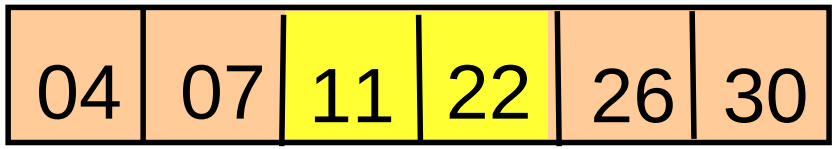

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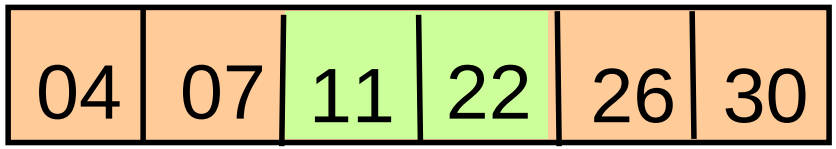

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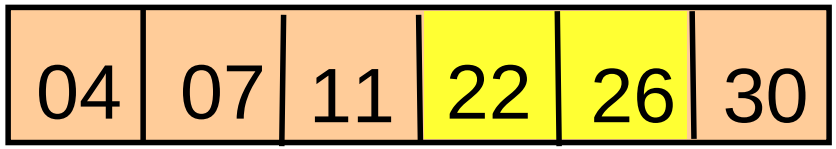

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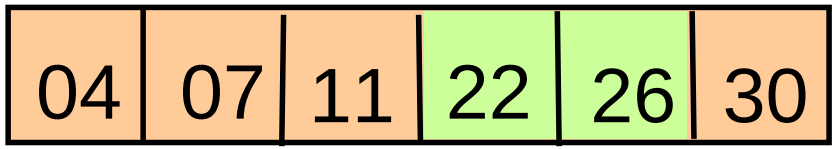

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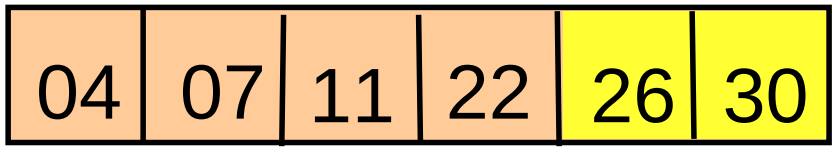

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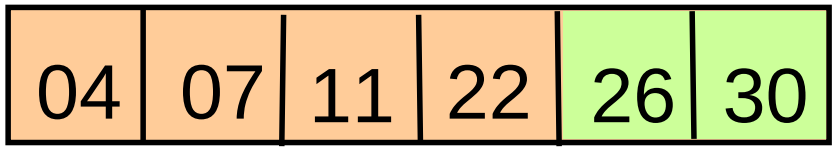

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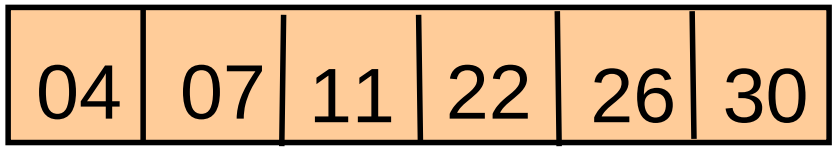

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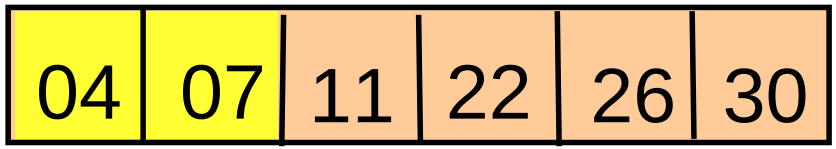
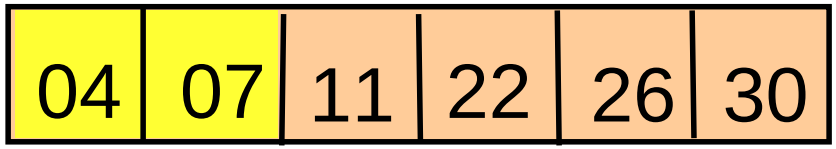


Bubble Sort

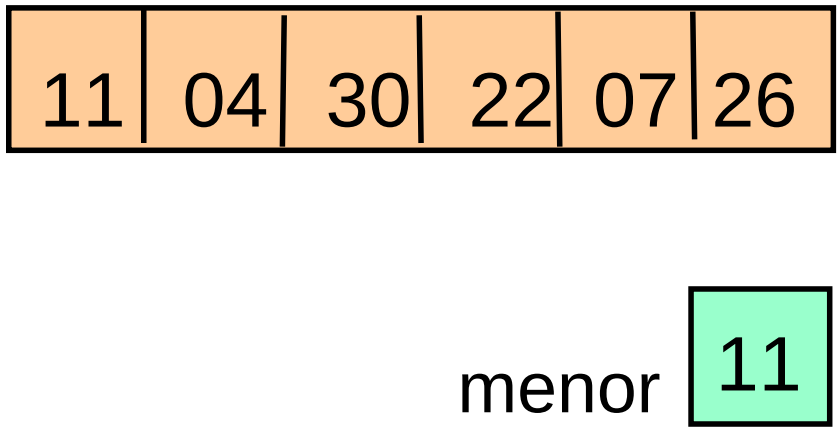


A cada iteração do laço mais externo do Bubble Sort, um número é colocado na posição correta, do maior para o menor.

Então, seu laço externo executa $n - 1$ iterações, para colocar todos os números em suas devidas posições.

Se não houver nenhum tratamento, o Buble Sort continua executando até que seu laço de repetição externo complete $n - 1$ iterações.

Selection Sort



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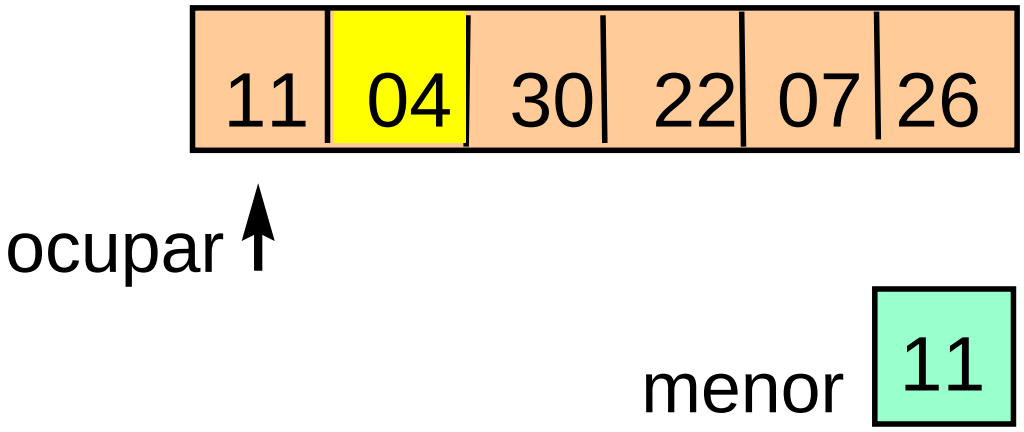

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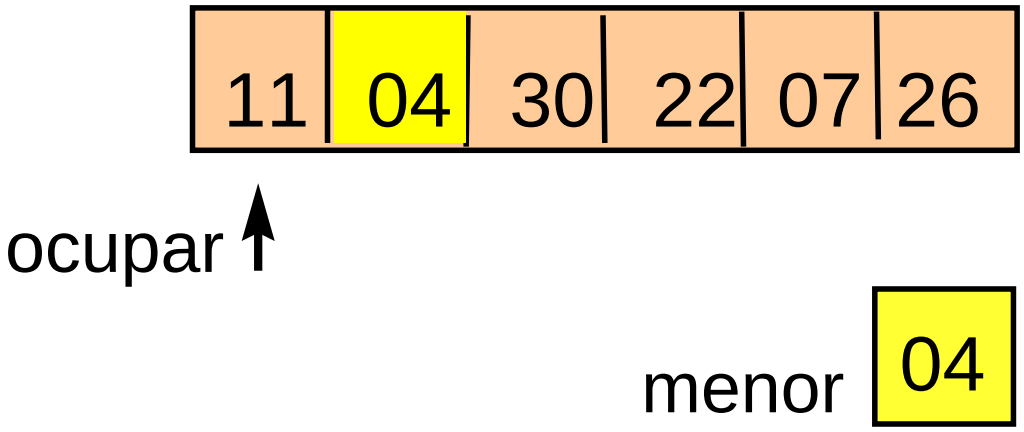

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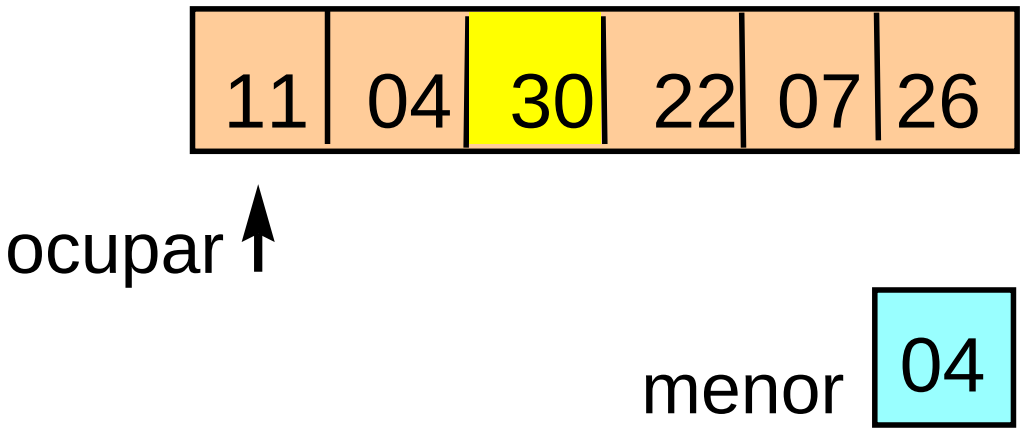

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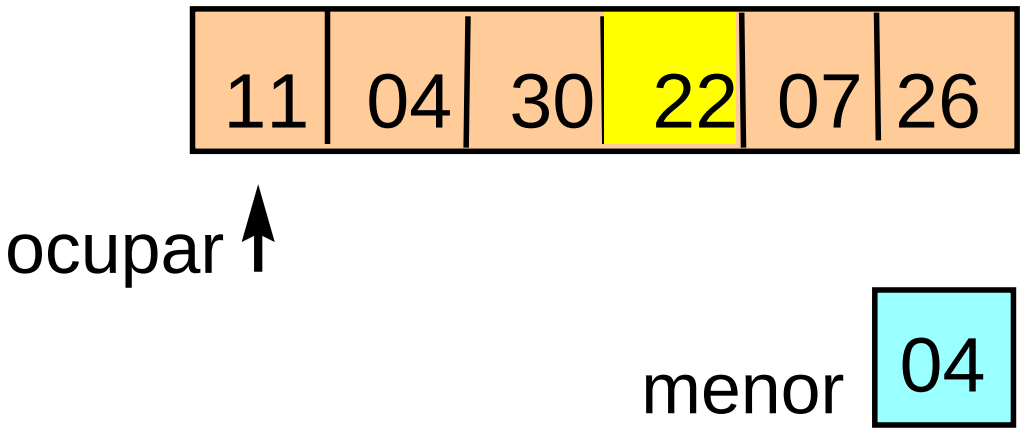

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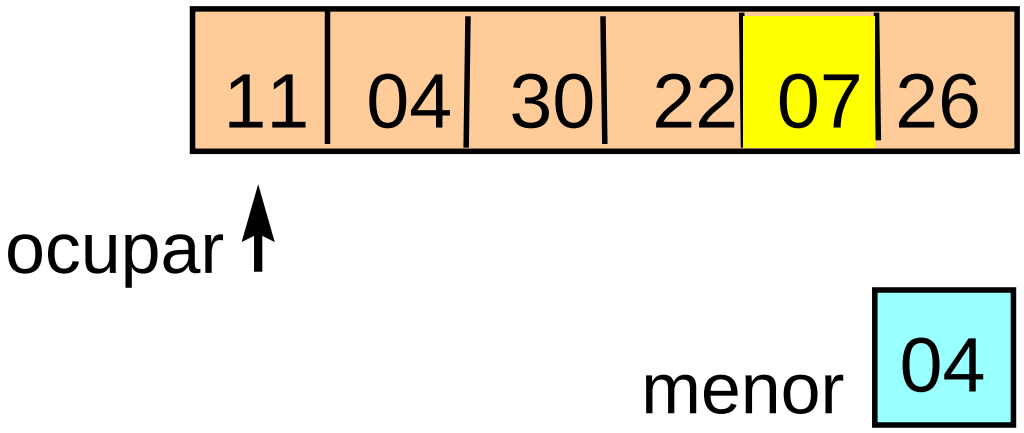

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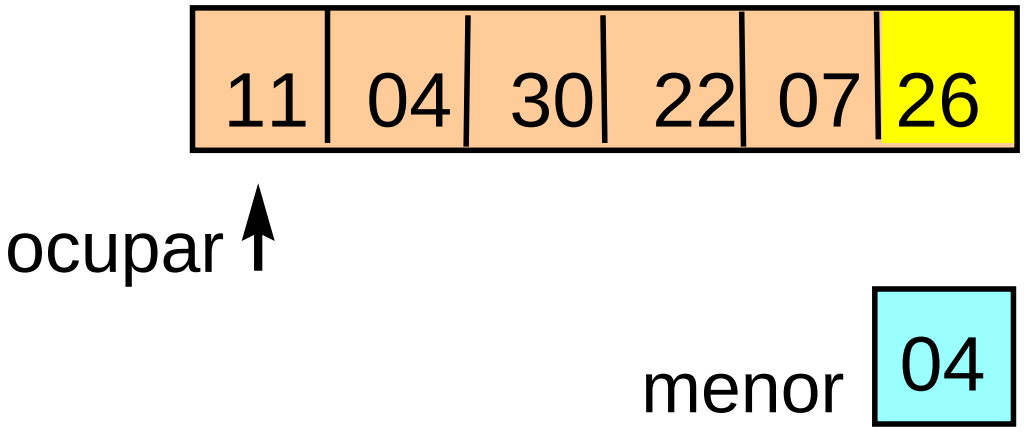

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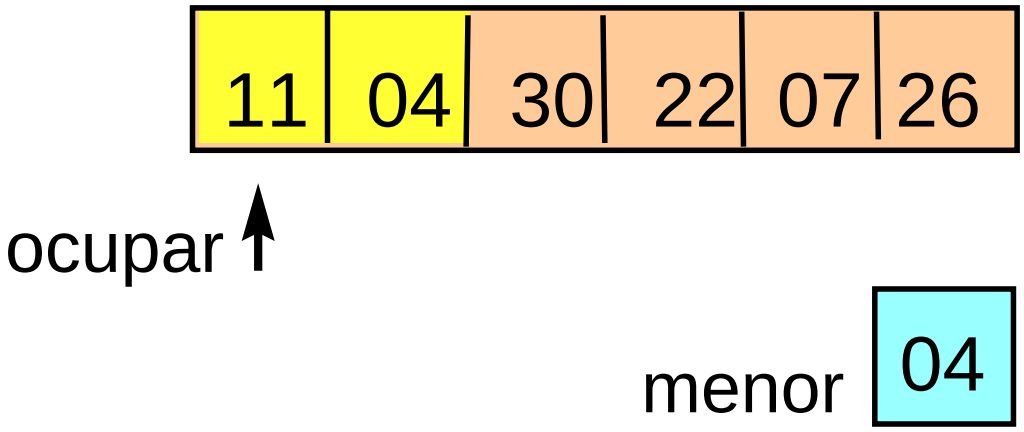

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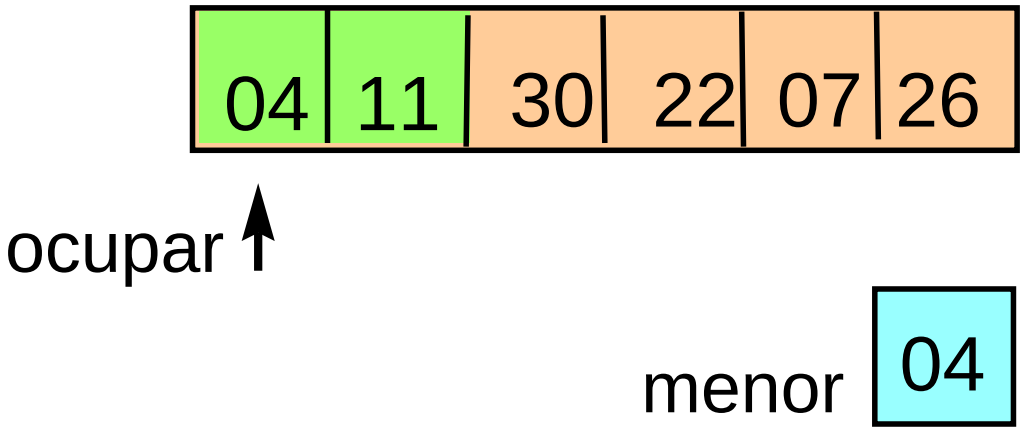
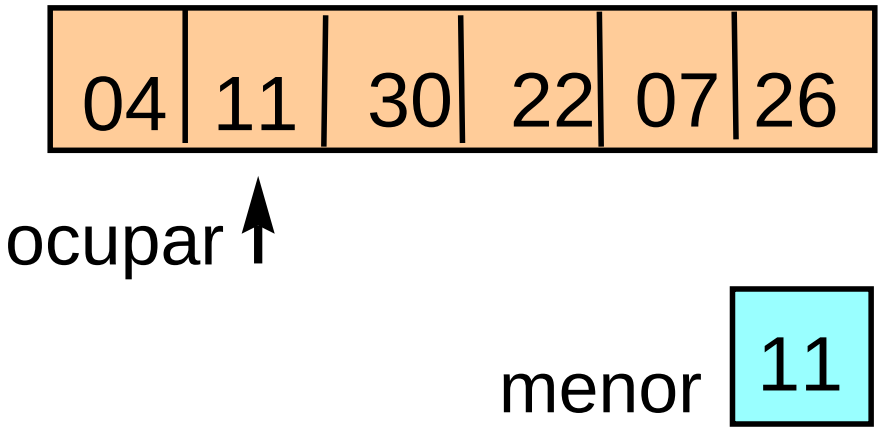


Selection Sort



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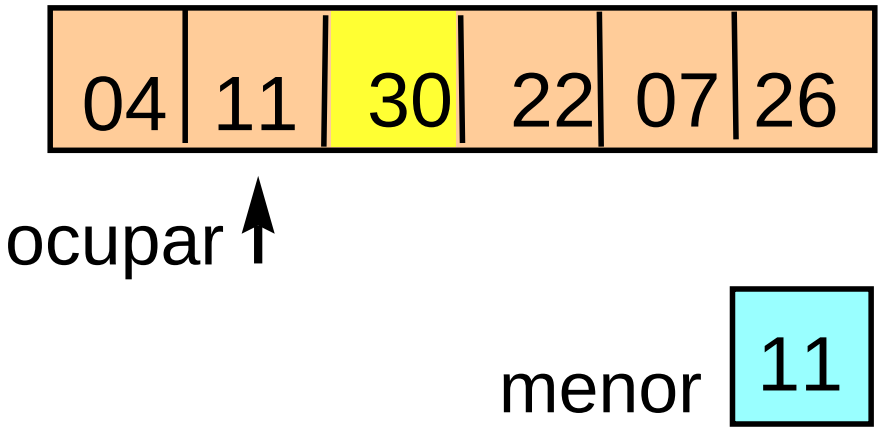

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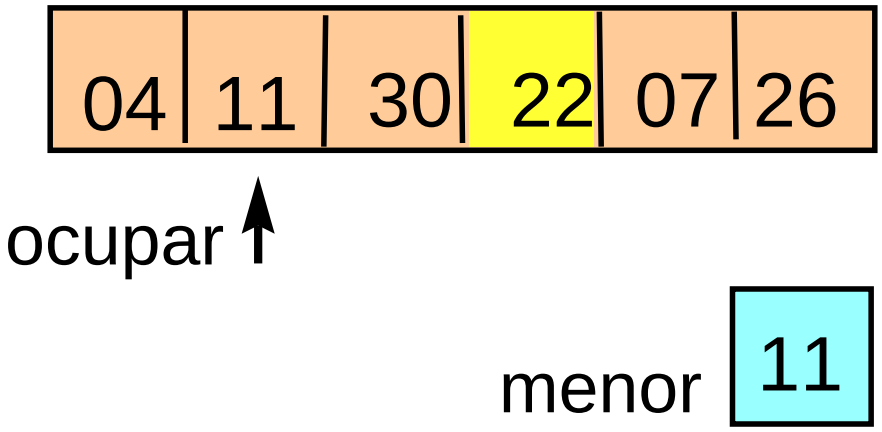

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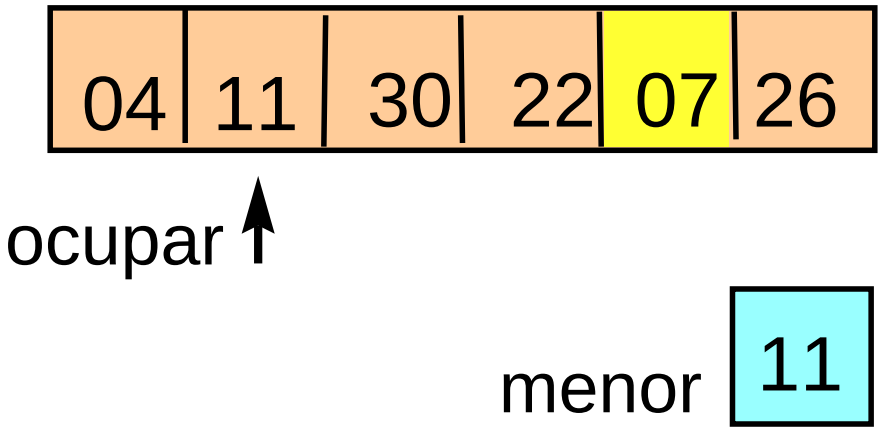

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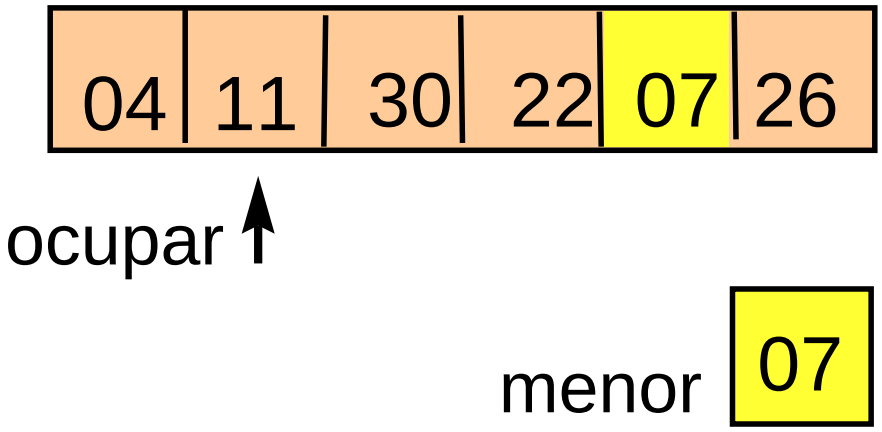

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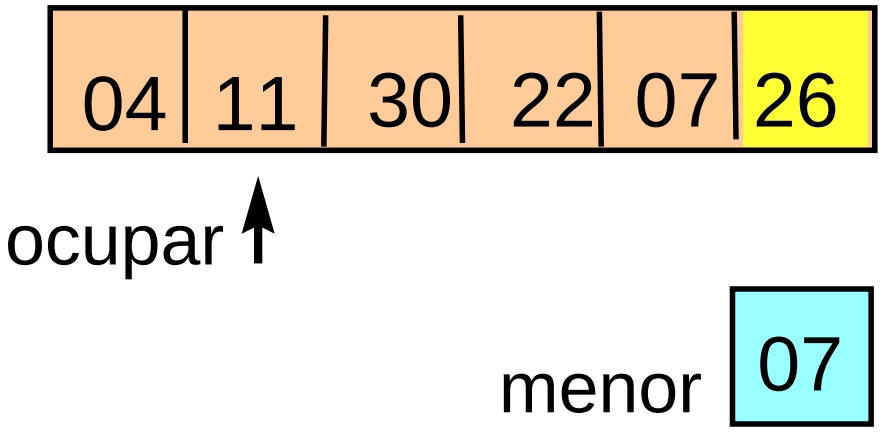

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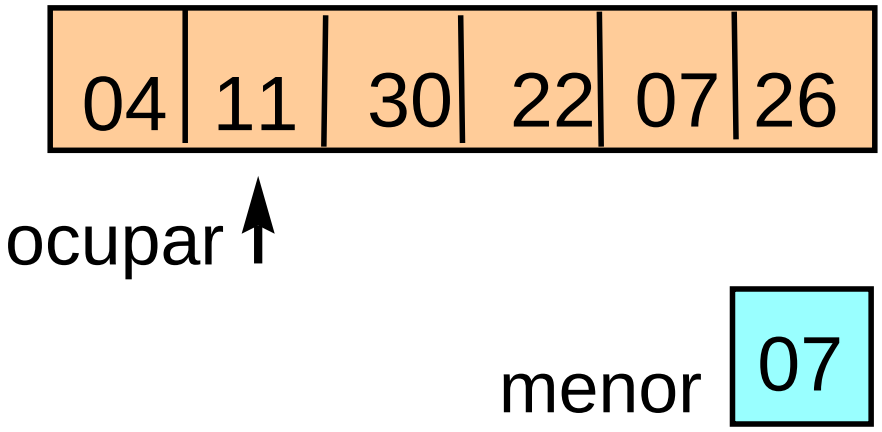

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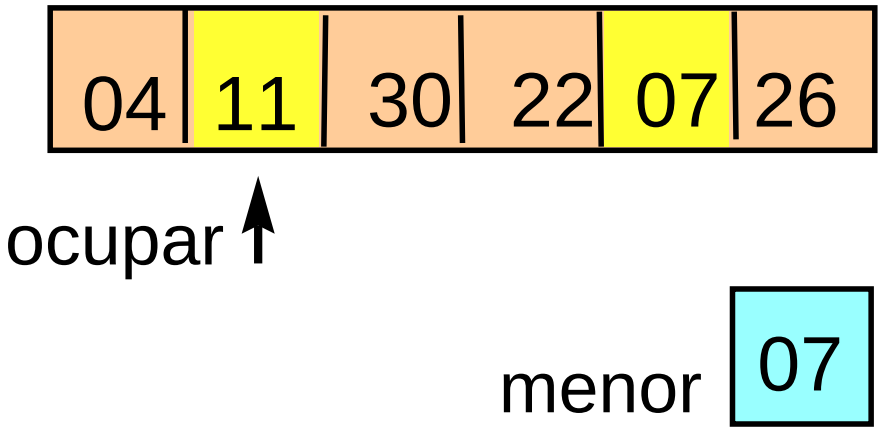

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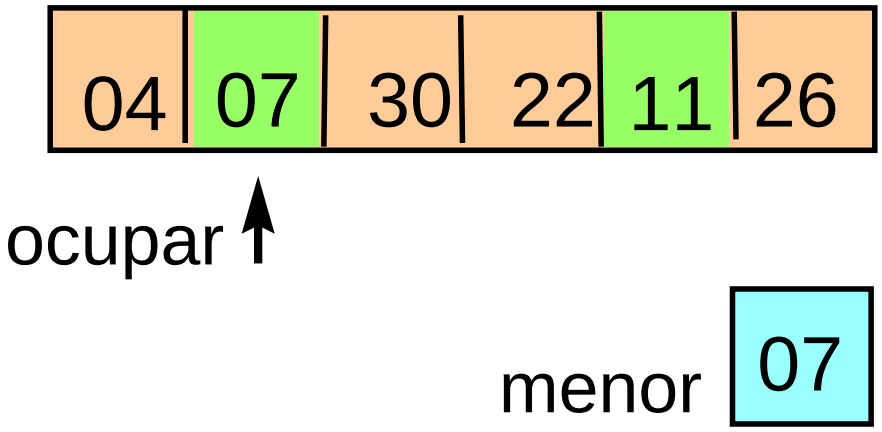

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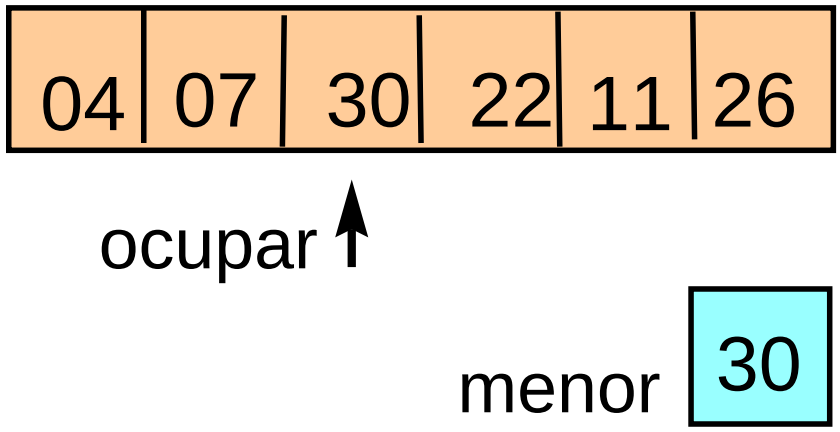

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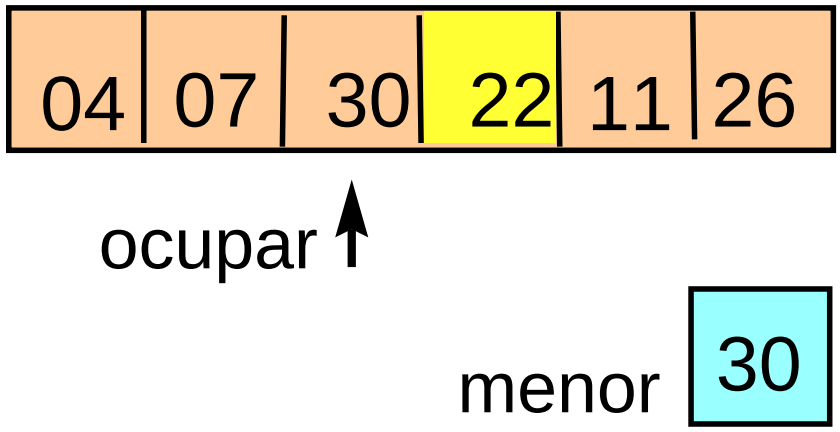

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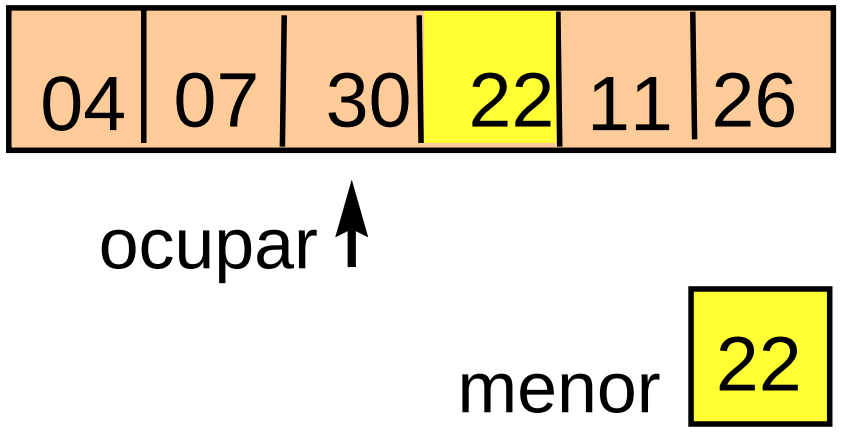

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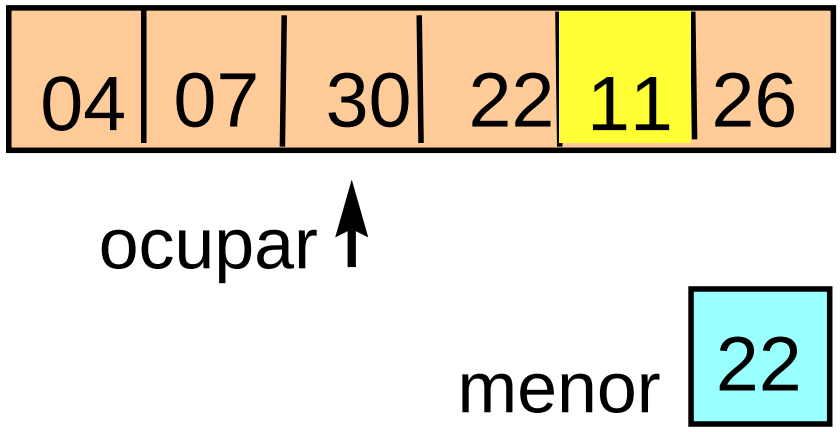

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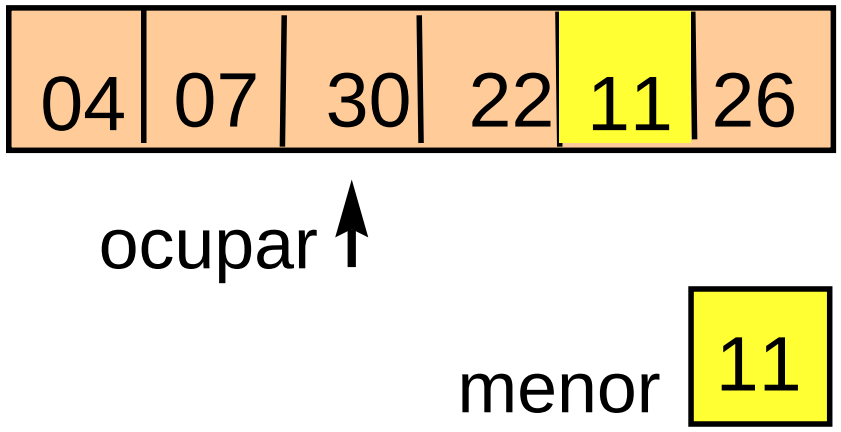

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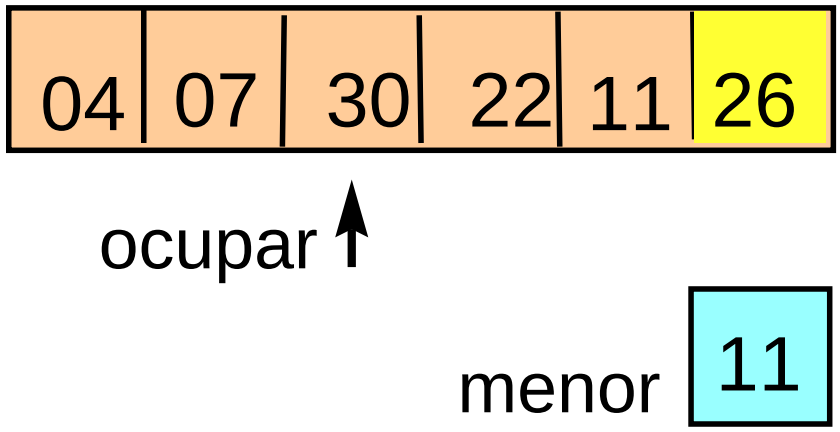

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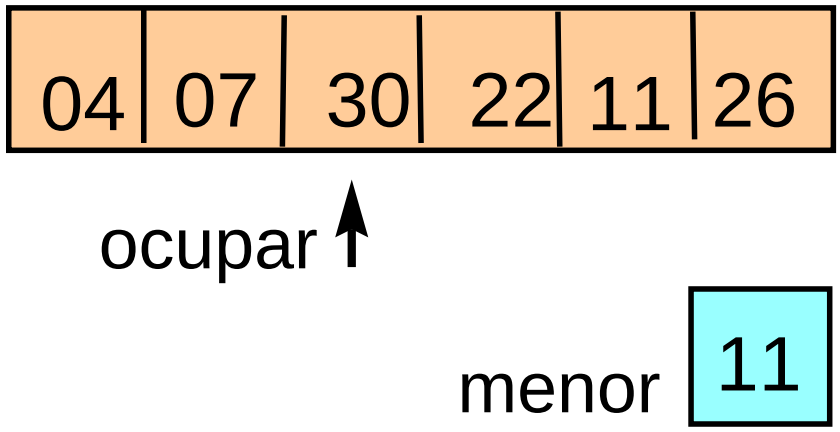

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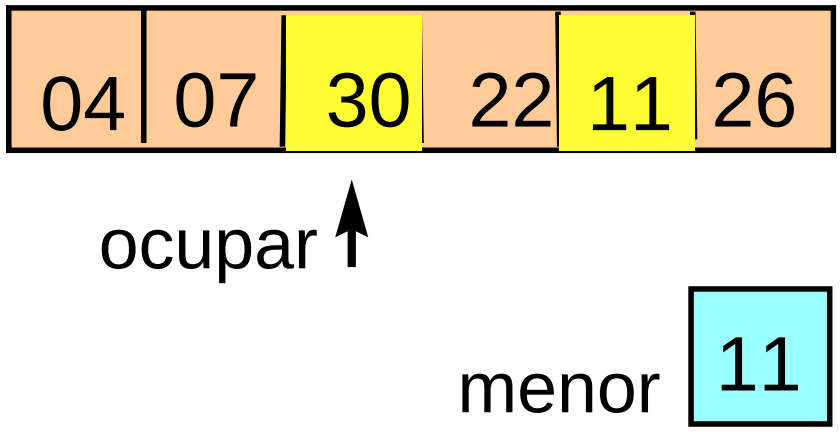

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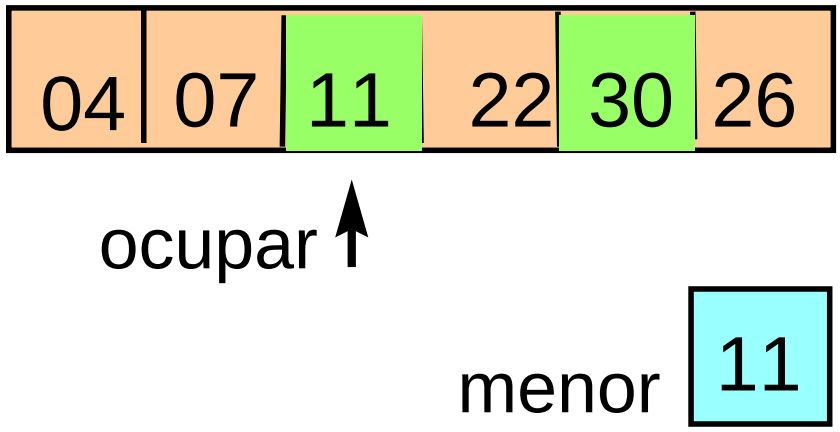

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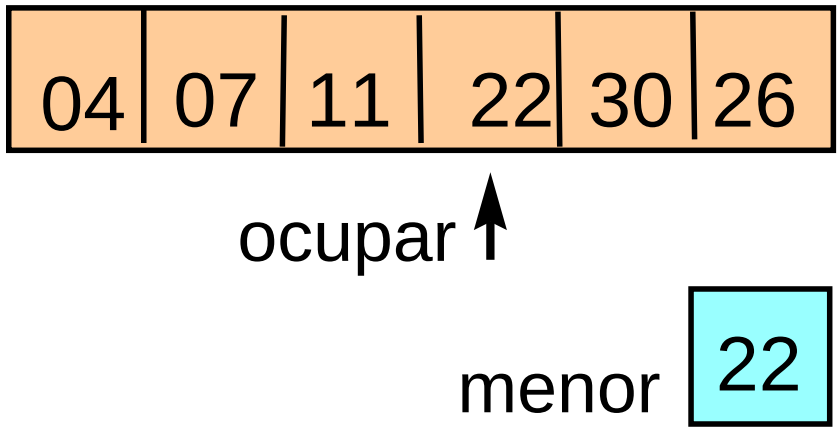

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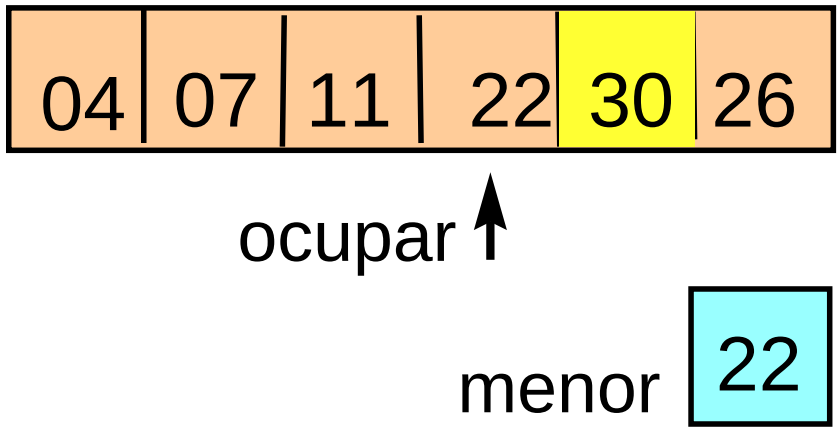

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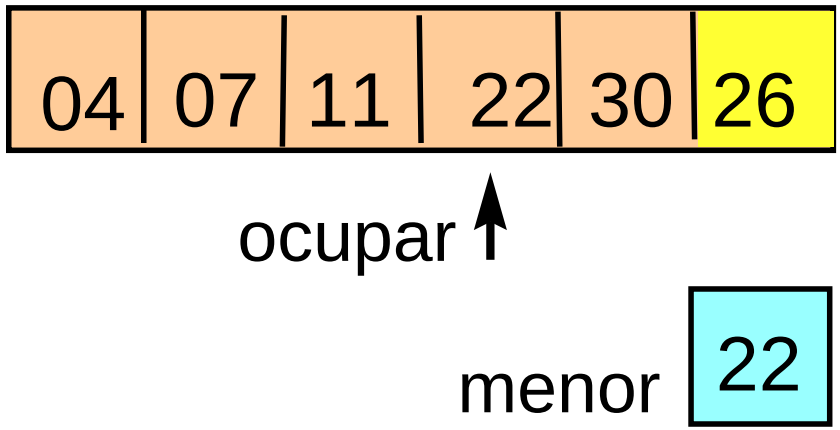

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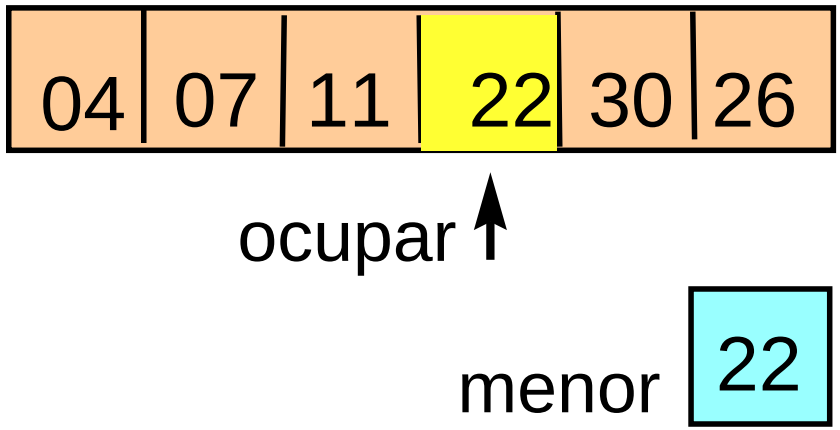

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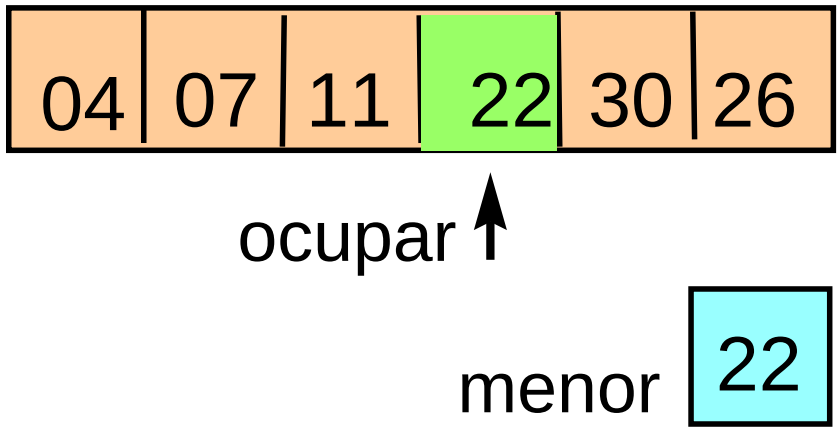

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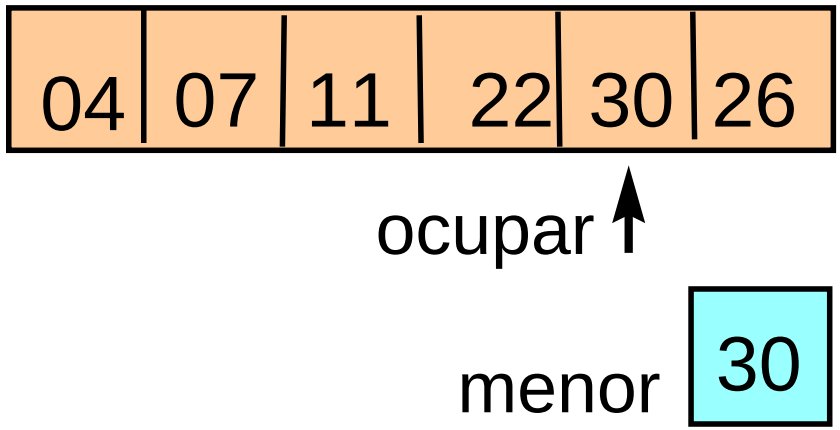

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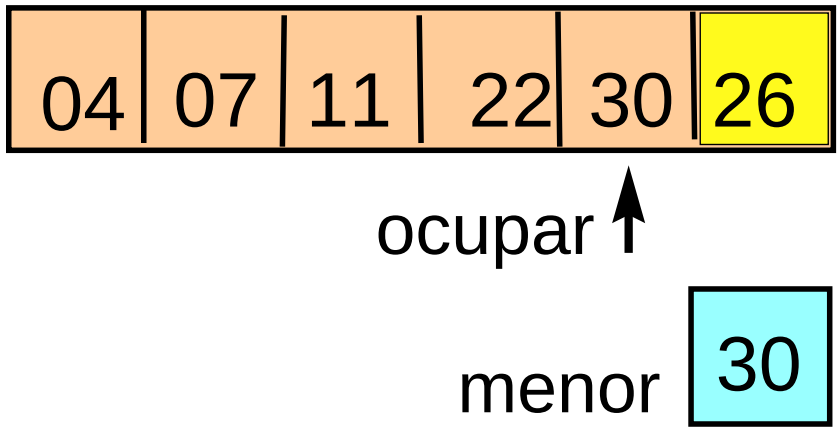

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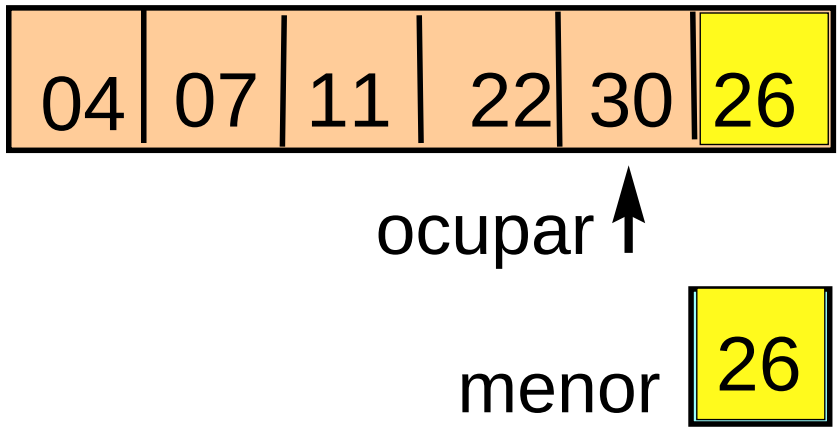

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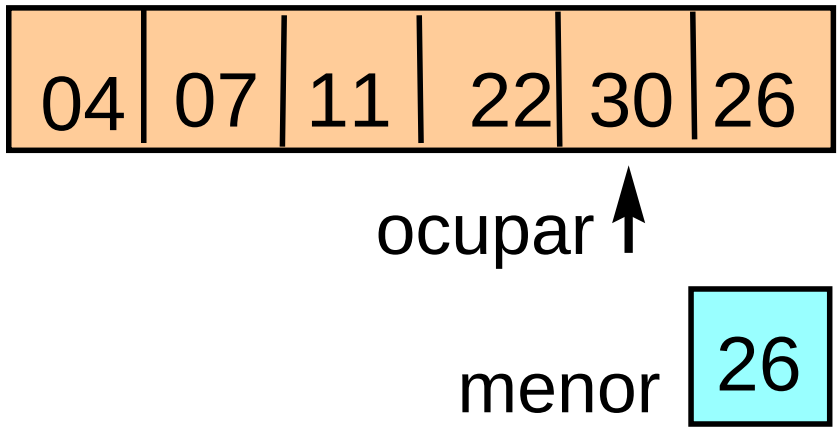

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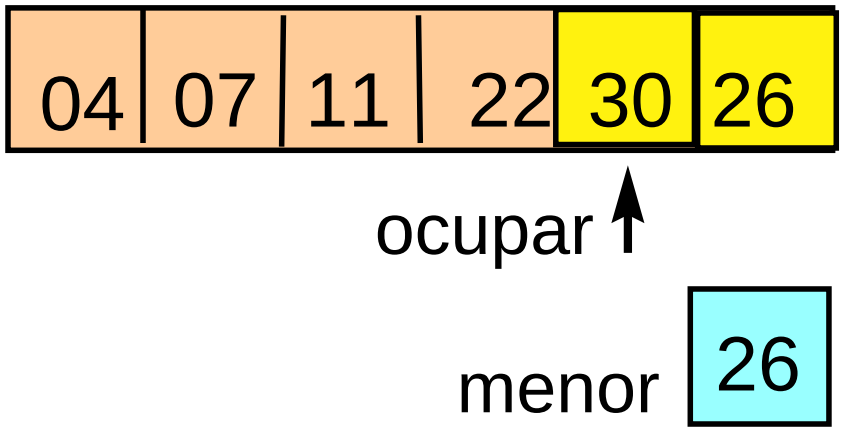

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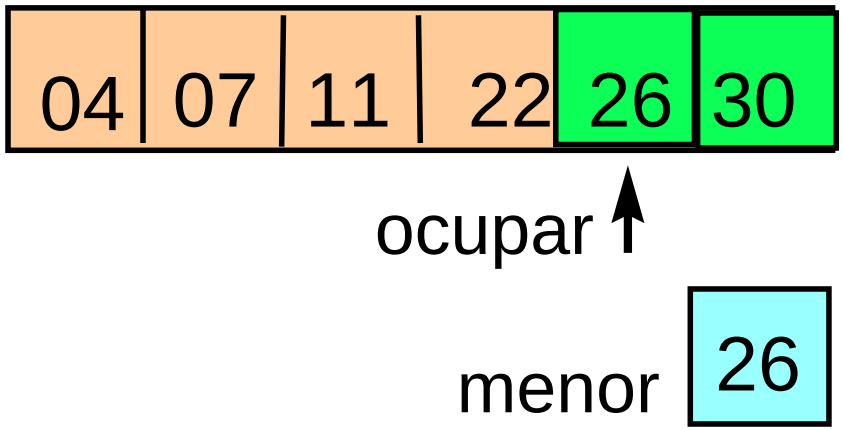

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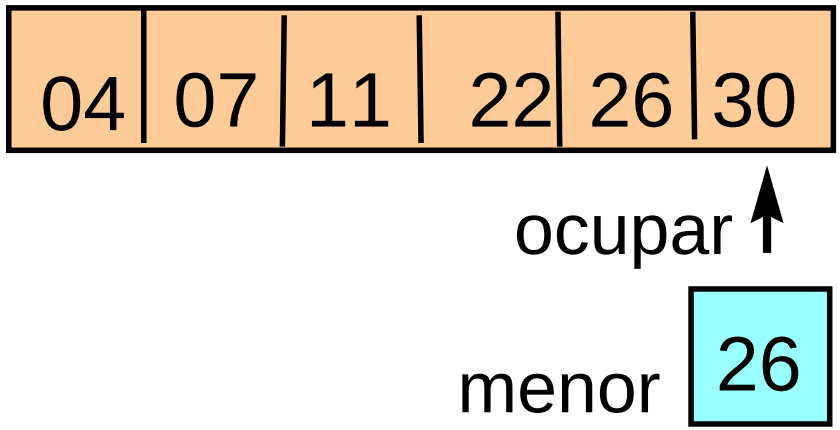

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